

```
In[1]:= (*Harmonic oscillator, exact solution of equations of motion*)
sol[φ0_, p0_, t_] = {φ[t], p[t]} /.
  DSolve[{φ'[t] == p[t], p'[t] == -φ[t], φ[0] == φ0, p[0] == p0}, {φ[t], p[t]}, t][[1]]
```

```
Out[1]:= {φ0 Cos[t] + p0 Sin[t], p0 Cos[t] - φ0 Sin[t]}
```

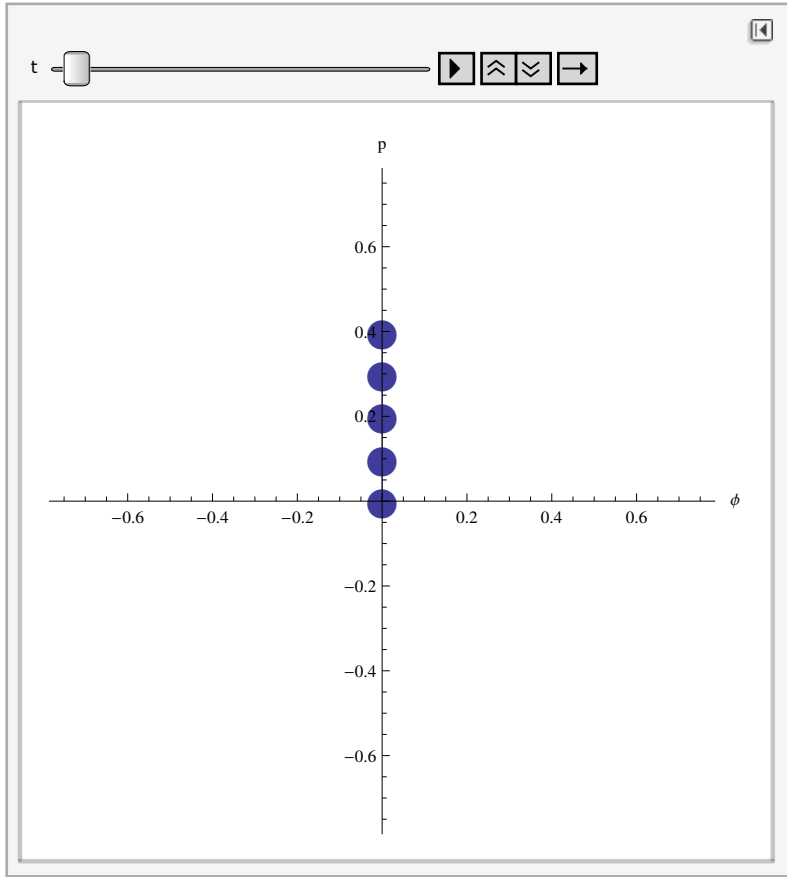
```
In[2]:= (*Pendulum, numerical solution*)
sol[φ0_, p0_] := NDSolve[{φ'[t] == p[t], p'[t] == -Sin[φ[t]], φ[0] == φ0, p[0] == p0},
  {φ, p}, {t, -10 π, 10 π}][[1]]
```

```
In[3]:= (*Initial conditions oscillator: [1] points [2] region *)
points[1] = {sol[0, 0], sol[0, 0.1], sol[0, 0.2], sol[0, 0.3], sol[0, 0.4]};
points[2] = {sol[-.1, .4], sol[-.1, .5], sol[-.1, .6], sol[.0, .6],
  sol[.1, .6], sol[.1, .5], sol[.1, .4], sol[0, .4], sol[-.1, .4]};
```

```
In[4]:= (*plot range in units of π *) a = 1/4;

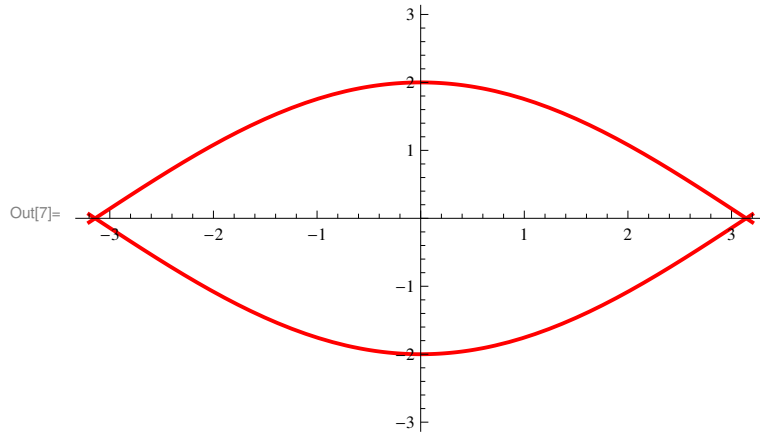
pointplot[t_, set_] :=
  ListPlot[{Evaluate[{Mod[φ[t], 2 π, -π], p[t]} /. points[set]]},
    PlotMarkers → {Automatic, Large}, AspectRatio → 1,
    PlotRange → a {{-π, π}, {-π, π}}, AxesLabel → {"φ", "p"}, Joined → True];
Animate[pointplot[t, 1], {t, 0, 2 π}, AnimationRunning → False,
  AppearanceElements → "ResetButton"]
(*change the argument <set>
to switch between the different initial conditions*)
```

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Out[6]=
```



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(*plot for longer times up till 8π to see
the anharmonicity appear for the outer points*)
```

```
In[7]:= separatrix = Plot[{2 Cos[t / 2], -2 Cos[t / 2]}, {t, -3.2, 3.2},
  PlotRange → {-π, π}, PlotStyle → {{Red, Thick}, {Red, Thick}}]
```



```
In[8]:= piplot[s_] := ParametricPlot[{{s1 π, t π}, {s1 π, t π}} /. s1 → s,
  {t, -1, 1}, PlotStyle → {Thick, Dashed, Orange};
  (*Show[{piplot[1], piplot[-1]}, PlotRange → {-π, π}]*)
```

```
In[9]:= stream = StreamPlot[{p, -Sin[φ]}, {φ, -π, π}, {p, -π, π},
  Axes → True, StreamScale → Medium, StreamStyle → Brown];
```

```
In[10]:= (*more initial conditions around the separatrix: [3] points [4] region *)
points[3] = {sol[0, 1.9], sol[0, 1.95], sol[0, 2.0], sol[0, 2.05]};
points[4] = {sol[-.1, 1.9], sol[-.1, 2.0], sol[-.1, 2.1], sol[.0, 2.1],
  sol[.1, 2.1], sol[.1, 2.0], sol[.1, 1.9], sol[0, 1.9], sol[-.1, 1.9]};
```

```
In[12]:= (*change value for a to 1 to see a larger portion of phase space *)
a = 1;
```

```

Animate[Show[{stream, piplot[1], piplot[-1], separatrix, pointplot[tmax, 3]},
  PlotRange -> (a + 0.1) {{-π, π}, {-π, π}}, {tmax, 0, 2 π},
  AnimationRunning -> False, AppearanceElements -> "ResetButton"]

```

Out[13]=

